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RABBI BEN EZRA AND THE HINDU-ARABIC PROBLEM.

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When Browning put in the mouth of Rabbi ben Ezra the words

"The Future I may face now I have proved the Past,"

he wrote better than he knew, for no scholar of the twelfth century had proved the Past more thoroughly than he, and few could face the Future with greater confidence.[1] Born, very likely at Toledo, between 1093 and 1096,[2] he became known as one of the most learned men of his time, and was the author of numerous books, a traveled scholar, a poet, and a man of great influence. He died in 1167, probably either in Rouen or in Rome.

Of the many lines of research pursued by Rabbi ben Ezra, one of the most interesting to students of the history of mathematics is that relating to the introduction of Hindu astronomy and computation into the Arabian civilization. Of that remarkable activity of the seventh century which resulted in the amalgamation of numerous semi-nomadic tribes into one mighty empire we have abundant knowledge; of the opening of the golden age of Mohammedan civilization and of its development under the caliphs of Bagdad in the eighth and ninth centuries we have well-authenticated records; of the influx of the Greek civilization through the translation of the classics of Alexandria and Athens we have the witness of a large number of manuscripts in the great collections of Europe and

[1] In the text of this article, the name Rabbi ben Ezra will be used because it is familiar to English readers. Properly, the name should be written Abraham ibn Ezra, although it often appears in the Latin translations as Abraham Judæus. In the notes we shall use the form Ibn Ezra.

[2] For a discussion of the matter, see Steinschneider, "Abraham Ibn Esra" in the *Abhandlungen zur Geschichte der Mathematik*, III, 59, Leipzig, 1880.

America; but of the details of the introduction of the Hindu mathematics into the region of Mesopotamia we are still in need of further information. To be sure, we have some recent light upon the subject through the writings of Severus Sebokht, a religious scholar of the seventh century,[1] but the problem is still far from solution.

Among the early sources that throw some light upon the situation, and which are as yet unknown to most European and American scholars, is Rabbi ben Ezra's account of the origin of Arab science, given in the introduction to his translation into Hebrew of a book written by Muhammed ibn Aḥmed el-Bîrûnî (973–1048)[2] on the astronomical tables of Mohammed ibn Mûsâ al Khowarizmi.[3] Although written in Arabic, this work of el-Bîrûnî is known only through two Hebrew manuscripts of ben Ezra's translation, one in the Bodleian Library[4] and the other, with the introduction by Rabbi ben Ezra, in the library at Parma.[5] The Hebrew title of the treatise is Ta'amê Lûchôth al Chowârezmî, and a portion comprising the introduction and the opening paragraphs of the book itself was published by Steinschneider in 1870,[6] with a German translation of the major part of the text.

Inasmuch as only a part of this important work has appeared in any modern language, and that in German, it has seemed desirable to translate all of the fragment from the Hebrew text, this being the part that relates to the introduction of Hindu mathematics among the Arabs. In so doing, we shall make use of Steinschneider's German translation only for purposes of comparison in the notes.[7]

As to the historical accuracy of Rabbi ben Ezra's statements in the introduction to his translation, it has been said that it is doubtful if he ever visited

[1] See J. Ginsburg in the *Bulletin of the American Mathematical Society*, Vol. 23, page 366; L. C. Karpinski in *Science* for June 12, 1912; E. R. Turner in *Popular Sci. Monthly* for December, 1912.

[2] Except in the case of names which have fairly well-recognized English forms, like al Khowarizmi, the transliteration followed is that of Suter's list in the *Abhandlungen zur Geschichte der Math.*, Bd. XI, Leipzig, 1900.

It was formerly supposed that this author was one el-Mutanâ of whom nothing further is known. Suter, however, corrected the erroneous impression by showing (*Bibliotheca Mathematica* (3), IV Bd., 1903, p. 125) that el-Bîrûnî and el-Mutanâ are the same. Steinschneider (*Orientalistische Literaturzeitung*, 1903, p. 486) explains the mistake as due to some copyist having combined the *beth* and *yod* and also the *resh* and *van* in the name el-Bîrûnî (אלבירוני), thus obtaining el-Mutanâ (אלמתני).

[3] The Mohammed ben Musa, whose algebra, translated into English by Rosen, was published in 1831. As given in Suter's list the name is Muhammed ibn Mûsâ el Chowârezmî (or Chwârezmî), Abû' Abdallâh. See also Suter's introduction to the new edition of al Khowarizmi's tables (Copenhagen, 1914).

[4] Mich. 400; No. 2006 in Neubauer's list of Hebrew MSS. in the Bodleian Library.

[5] Rossi 212. The transcription of this manuscript used by Steinschneider is now in the library of the Jewish Theological Seminary of America, New York.

[6] "Zur Geschichte der Uebersetzungen aus dem Indischen ins Arabische," in the *Zeitschrift der deutschen Morgenländischen Gesellschaft*, Bd. XXIV, p. 325.

[7] Use is also made of the notes by Steinschneider and of the material given by Suter, *l. c.*, and by Wüstenfeld in his *Geschichte der arabischen Aerzte*, Göttingen, 1840. A number of helpful suggestions have also been made by Professor Alexander Marx of the Jewish Theological Seminary of America, New York City.

India,[1] but this would not be at all necessary. He was as careful a writer as any of his time, a scientist of high repute, a student of the history of sciences, and a man less given to the acceptance of mere tradition than was usually the case. It is to be expected from his reputation that he would have consulted the best Arabic sources available, although we have at present only slight knowledge of what those sources were, and although no manuscripts thus far translated throw any light upon the problem.

The following is a translation of the introduction written by Rabbi ben Ezra:

"In the name of the Most Holy and Revered, in whose help I trust, spake Abraham ibn Ezra the Spaniard. In ancient times there was no wisdom and no [true] religion among the sons of Ishmael, the tent dwellers, until the [author of the] Kora[2] came and gave to them from his heart a new religion.

"After him there appeared many sages among them, who wrote many books on their laws; but at last there appeared a great king in Ishmael, called e's Saffah,[3] who heard that there were many sciences in India. And he gave orders to search for a scholar who should know the language of India and that of Arabia, so as to translate for him one of their books of wisdom, although he feared that a calamity might befall him,[4] since profane sciences [were then permitted] in Ishmael in the book of the Koran alone, and whatever of the sciences they received [by tradition] was [believed to be] therein. [He had heard that] in India there was a book, very important in the councils of the kingdom, that was arranged in the form of stories put in the mouths of dumb creatures, the large number of pictures rendering the book very valuable in the eyes of the reader. And the name of the book was Kalilah we-Dimnah,[5] which means the Lion and the Bull, because the first gate[6] of the book refers to them.[7] And the above named king fasted forty days, hoping to see the angel of dreams who should

[1] Steinschneider adduces a proof that Ibn Ezra did not visit India. See *Zeitschrift der deutschen Morgenländischen Gesellschaft*, Bd. XX, p. 430.

[2] For *koran*.

[3] In the De Rossi codex this name appears as Altsaphak; but evidently the Abbasside el-Saffah is meant, and this fixes the time as the middle or second half of the eighth century. A similar story is told of Nushirvan the Just, who reigned in Persia in the sixth century.

[4] That is, the translator. This calamity might befall one who assumed, by translating such a work, that the Koran was not all sufficient. In the Hebrew a few words are omitted but this seems to be the sense of the passage.

[5] Called by the Hindus the Kurtuk Dumnik.

[6] That is, the first chapter.

[7] Although the first chapter (in some editions the second) of the Kalilah we-Dimnah deals with the bull and the lion, the name of the book is derived from the names of two jackals, Karattaka and Damanaka, which play an important part in the stories of the subsequent chapters. See Theodor Benfey's introduction to G. Bickel's edition of the Syriac *Kalilag und Damnag* (Leipzig, 1876, page 11); Wollaston's English translation under the title *Lights of Canopus* (London, 1904); J. Derenbourg, "Kalîlâh et Dimnâh" in the *Bibliothèque de l'école des hautes études* (Paris, 1881). The first edition appeared in Johannes de Capua's *Directorium humanae vitae*, "Et vocatur liber Kelile et dimne," Strassburg, c. 1488–1493.

In the Talmud, mention is made of Ben Tiglah and Ben Laanah, and the relation of these to the Kalilah we-Dimnah was shown by Wolf, in the *Bibliotheca Hebraea* I, 932, note. See also Steinschneider, in *Jewish Literature*, London, 1857 (p. 279, note 54, *a*). Geiger, in the *Jüd. Zeitschrift*, VII (1869), 139, sets forth his belief that Ben Tiglah and Ben Laanah are the names of two Jewish authors.

allow him to translate the book into Arabic. Then he had a dream in harmony with his thoughts.[1] He thereupon sent for a Jew who lived in his time and who knew the two languages, and he gave him command to translate the book, since he feared that if an Arab should translate it he might die.[2] And when he saw how wonderful the book was, and so it really is, he was overcome by a desire to know more. Then he gave great wealth to the Jew so that he might journey to the city of Arin[3] on the equator, under the signs of the Ram and the Scales, where the day throughout the year is equal to the night, [thinking '] Perhaps he will succeed to bring one of their wise men to the king ']. And the Jew went [there] and indulged in many subterfuges, after which, for a large sum, one of the wise men of Arin agreed to go to the king, and the Jew swore to him that he would not detain him beyond a year and that he would return him to his home. Then this scholar, whose name was KNKH,[4] [was taken to the king] and taught the Arabs the bases of number, that is, the nine numerals.[5]

"From the mouth of the learned man, through the Jew [as] an Ishmaelite interpreter, a scholar by the name of Jacob ibn Sharah[6] translated the book of the tables of the seven planets and the creation[7] of the earth,[8] the degree of rise, the establishing of the houses,[9] and the knowledge of the upper stars and the darkening of the lights.[10] No explanation of these matters was set forth in the book, only operations in the form of rules to be accepted on faith.[11] The average motion of the planets was computed according to Hindu methods, their cycle, called Hazervan, being equal to 432,000 years.[12]

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[1] This seems to have been looked upon as a sanction for the translation of profane works, for el Mâmûn waited until he saw Aristotle in a dream before ordering the translation of the Greek philosophers.

[2] That is, an Arab might be punished for the profanity of such a translation while a Jew might escape.

[3] Arin seems to have been an astronomical center of India, like Ujjain. Indeed, Reinaud thinks the two places were the same. Al Khowarizmi's tables were constructed on the basis of this meridian. See Suter's edition of the tables, *l. c.*

[4] In the Hebrew text the vowel points are not given. The name may be Kanka or Kanaka; for, according to Meǧriti (959), Kanka was the inventor of amicable numbers.

The subject is deserving of a more extensive report than is possible in this connection.

[5] Evidently the decimal system, else "bases of number" would have no significance.

[6] Since in the Spanish rabbinical script the Hebrew letters *shin* and *theth* are easily confused, as also the letters *he* and *gaf*, a scribe who was not familiar with the name might easily have written Sharah for Tarik. Hence this Jacob ibn Sharah may have been the same as the famous astronomer and astrologer Ja'qûb ibn Târiq mentioned by el-Bîrûnî as having been living in 777.

[7] Literally, "the whole work."

[8] Was this the Sûrya Siddhânta, the great astronomical work of the Hindus, written in the fourth century?

[9] That is, in the scheme of astrology. The passage is very obscure, and several words omitted at this point in this translation seem to have referred to the ascension and declination of the planets. Steinschneider was also unable to find the exact meaning of the passage.

[10] Eclipses of the sun and moon.

[11] The Hindus gave no proofs of their propositions, as may be seen by examining the works of Âryabhata, Brahmagupta, Mahâ îr, and Bhaskara, all of which are now accessible in translation.

[12] In the Sûrya Siddhânta is this passage: "To determine the saura years contained in this aggregate, write down the following numbers, 4, 3, 2, which multiply by 10,000; the product, 4,320,000, is the aggregate or Mahà yuga. . . . Divide the aggregate 4,320,000 by 10 and multiply the quotient by 4 for the satya yug." See *Asiatic Researches*, 1790, II, p. 230.

"After [the death of] Jacob the translator there arose a great scholar in Ishmael who knew the secrets of the wisdom of counting [and of] chronology and [who] reduced the average planetary motion to the era of Ezdeger,[1] the last of the Persian kings, for the Arabs conquered the kingdom of Persia and converted the inhabitants to their own religion. This scholar was Mohammed ibn Mûsâ al Khowârezmî,[2] and all later Arabic scholars do their multiplications, divisions, and extraction of roots as is written in the book of the [Hindu] scholar which they possess in translation.[3] He[4] prepared, in a more convenient form for students, tables which were the equal of the work of KNKH, but he gave no explanation of his statements. After him there arose in Ishmael[5] a scholar called el-Ferġânî,[6] who set forth reasons for the words of al Khowarizmi. After el-Ferġânî[7] another great scholar translated a very important book about the stars, written by Ptolemy, king of Egypt,[8] a Greek who lived a thousand years ago.[9] This book[10] is perfect, there being nothing higher in the science of the spheres, their secrets, their distances from the earth,[11] and the measure of the upper stars in the sphere of the zodiac. He divided them[12] into six classes, of which the first [class] was called the first glory. He established the number of stars in each class and enumerated all [the stars] in the forty-eight constellations,[13] namely, the constellations of the entire sphere which contains 1,022 stars besides the clouded ones.[14] He gave reasons for all corrections[15] and, in general,

It is interesting to note that the number 432,000 also appears in the Babylonian chronology. According to Berossus, a Babylonian priest (250 B. C.), the antediluvian kings of Babylon reigned for a total period of 432,000 years. See W. H. Roscher, *Die Zahl 40 im Glauben, Brauch und Schrifttum der Semiten*, Leipzig, 1909, p. 97; H. V. Hilprecht, *Mathematical . . . Tablets from . . . Nippur*, Philadelphia, 1906, p. 21.

[1] This begins with the day when Jezdegird III ascended the throne of the Sassanides, June 16, 632, of the Christian calendar of that period. Although the Mohammedan conquest soon imposed the calendar of Islam on most of Persia, the Jezdegirdian calendar survived among the Persians in the southern provinces and in western India.

[2] Mohammed the son of Moses, the Kharezmite, who flourished in Bagdad early in the ninth century and whose *Algebr w'al-Muqabala* gave the name to algebra.

[3] It was probably this work which al Khowârizmî made the basis of the treatise on computation which, as is well known, was translated into Latin in the twelfth century under the title *Liber Algorismi* (Book of al Khowârizmî), from which we have the word *algorism*.

[4] Al Khowârizmî.

[5] In the original, Israel, probably a mistake of the scribe.

[6] Ahmed b. Muh. b. Ketîr el-Ferġânî or el-Ferġûnî was one of the astronomers and followers of el-Mâmûn.

[7] In the original, "after him."

[8] In the Middle Ages, and even later, Claude Ptolemy (second century A. D.), the great astronomer, was commonly mistaken for one of the kings who reigned under the name of Ptolemy.

[9] This was approximately correct, since Ibn Ezra lived in the twelfth century, while Ptolemy lived in the second century.

[10] The Almagest.

[11] The original may also be translated as motion with respect to the earth.

[12] The stars.

[13] Literally, "figures."

[14] Ptolemy's tables, edited by Peters and Knobel (Washington, 1915), give 48 groups or constellations and 1028 stars. The number 1,022 is not only given by Ibn Ezra but is also given by el-Ferġânî in his compendium of the Almagest, as shown in a Hebrew MS. of the latter's compendium now in the Columbia University Library (X 893, T 522). This shows that the number in Ibn Ezra's work was not an error of some scribe. Moreover, in the Manitius edition (1912) of the Almagest the number is given as 1,022.

for everything found in the tables translated by Jacob from the mouth of the Indian scholar.

"All the proofs given by Ptolemy or Talmi in his great book Almagest are perfect, and no man can contradict them, for these are proofs from the science of magnitude or the science of measurements, which is called in the Greek tongue Yeometria and, by the holy sages of Israel, Gematria.[1]

"On this book[2] commentaries were written by many sages in Ishmael, the most distinguished among them, in mathematics and astronomy, being the scholar Muhammed ibn Mutani.[3] He [it was who] compiled for one of his family a very important book about the corrections[4] of the planets and the explanation of the contents of the tables in the book of al Khowarizmi, and briefly mentioned the proofs and illustrations, their principles being taken from the book [called] Almagest. In certain passages, however, his explanations are more complete than those of King Ptolemy, and in these are also places where he sets forth mathematical proofs [which were] invented by himself.[5] In most places he agrees with the theories of el-Fergânî who was mentioned above. For the sake of clearness his book is arranged in the form of questions and answers.

"Said Abraham [ibn Ezra]: Except in a few places there is no difference between the norms[6] of the planets as given in Ptolemy's work and those of the Hindu scholar, and at the proper time I shall mention how this difference arises. I have written a book on the mean motion of the planets, and on the head and tail[7] of the *tali*[8] as observed by the astrolabe, because the positions of the planets

[15] Steinschneider translates the Hebrew word *tikunim* as "norms," but the word also means corrections or interpolations, and this is much more in harmony with the context at this point, although in two later sentences the word "norms" seems more appropriate.

[1] This is not correct, for gematria was an entirely different science, in no way related to geometry. It is probable that the error is that of some copyist of Ibn Ezra's manuscript, although the latter may himself have been at fault. Without the vowel points the word can be read either geometria or gematria. Because of this fact the pronunciation of the word as it appears in the Talmud was quite unsettled until the nineteenth century. Ibn Ezra asserts elsewhere that the Almagest was translated into the Arabic after the time of el-Fergânî, and at any rate after the time of al Khowarizmi. This confirms the view, expressed by Weber (*Naxatra*, I, 321), that the Arabs were familiar with the science of the Hindus before they became acquainted with the works of Ptolemy. (See also Woepcke, *Sur l'introduction de l'Arithmétique indienne en occident*, Rome, 1859, p. 58, and Lassen, *Indische Alterthumskunde*, II, 1139.) Reinaud (*Abulfeda*, pp. XLI–XLIII) states that the Almagest was translated completely into Arabic under el-Mâmûn, but that it was translated into Syriac and Hebrew in the middle of the eighth century, a doubtful statement as to the Hebrews for the reason that they were not generally interested in such matters at that time. See Steinschneider in the *Zeitschrift der Deutsch. Morgenl. Gesellsch.*, XXIV, page 337.

[2] The Almagest.

[3] The Muhammed ibn Ahmed el-Bîrûnî referred to in the note on page 100.

[4] See the note on the Hebrew *tikunim*, *supra*.

[5] Literally, from his heart.

[6] See the note on the word *tikunim*, *supra*. We have taken Steinschneider's translation here, since it seems to make better sense than "corrections." What is apparently meant is "tables."

[7] In the Hindu writings, Rahu and Ketu.

[8] The *tali* was a celestial dragon believed by early writers to be the cause of eclipses and of various other disturbances in the heavens. The Chinese attributed eclipses of the moon to the fact that it was covered by the head or the tail of the dragon, and to prevent it from being devoured they tried to frighten the dragon away by the noise of cymbals and tambourines.

The word *atalû*, used by the Babylonians to mean eclipse, is evidently the source of the

in the tables of al Khowarizmi do not agree with their observed positions by $9\frac{1}{2}$ degrees. It is my opinion that the idea of the Hindu scholars as to mean planetary motion is based upon the representation on the plane, and this is correct according to the science of projection but not according to the science of astronomy.[1]

"The tables in Almagest are useless for the reason that they are evidently corrupted. Moreover, they are not in accord with the paths of the stars.[2] The errors are not due to Ptolemy, however, but to the ancients from whom he derived [his information], and this I shall discuss later when I shall have completed this work. The norms in my book are the same as Ptolemy's and as those used by all Arabian scholars. [The latter] prepared many tables and were more exact in their work than Ptolemy, and I shall hereafter explain the reason. Only the norm of the sun is not[3] the same as Ptolemy's, being less than that by 29 minutes, for he relied upon the observations of ABRKS[4] who lived 208 years before Ptolemy.[5] He could not have relied upon the testimony of Fitin and Afitimon[6] who lived about 1,000 years before him,[7] for they could not have made [as good] an astrolabe [as the one] used by Ptolemy. Hipparchus had stated that the position of the sun at its apogee[8] was in his time at 5 degrees of the Twins,[9] and since Ptolemy found it in nearly the same place he inferred that the position of apogee of the sun, unlike the positions of apogee of the five planets, does not change; but many scholars found that it changes as well as

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Syriac word *atalia* and the Hebrew *tali*, both of which signify dragon and are used in connection with eclipses.

This idea of the power of the *atalia* was ridiculed by Severus Sebokht in the seventh century, and of course Ibn Ezra had no illusions concerning any dragon.

See Morris Jastrow, *Religious Belief in Babylonia and Syria*, page 213; M. F. Nau in the *Journal asiatique*, 1910, ser. X, Vol. 16, page 219.

[1] Literally, "is according to the image of the galgal," etc. Galgal means a circle, and the passage probably means that the Hindus studied planetary motion from the representation of the heavens as circular, on a plane, rather than from a celestial sphere. The passage is more obscure than any other in the book, and Steinschneider was also unable to satisfy himself as to its exact meaning.

[2] Literally, "do not follow the way of the images." The passage is obscure, and may refer to the paths of the planets in the constellations or to the zodiac.

[3] In Ibn Ezra's work here described.

[4] That is, Hipparchus, who lived about 140 B. C. Query: is there any relation of Abrks to the Gnostic term abraxas?

Peters and Knobel, *l. c.*, page 7, agree with Delambre that the catalogue of stars in the Almagest is due to Hipparchus. This was Ibn Ezra's opinion.

[5] This is only approximately correct, since Ptolemy lived in the second century A. D. Probably the 208 is a scribe's error for 280, since Ptolemy himself speaks of Hipparchus as having lived 285 years earlier than he.

[6] That is, Meton and Euctemon, who flourished in the fifth century B. C. Ptolemy states (*Almagest*, III, 1, which is Vol. I, p. 141 in the translation of Manitius) that he had to omit all reference to the observations made by the school of Meton and Euctemon. On the names used by Ibn Ezra, see Isak Israel, Jesod Olam, IV, 7; *Mag. für die Literatur des Auslandes*, 1846, p. 378.

In the fragment of Levi ben Abraham's work on astrology (Cod. Reggio, 13, fol. 56, now Cod. Oxf. 2028) the names appear as Meton and Euctemon.

[7] Probably 600 years would be nearer the truth

[8] Literally, "Place of the high point of the sun."

[9] He found it to be 5° 30′ in the year 140 B. C.

in the case of the planets. Its position now, in the year 1160 of the era of the uncircumcised,[1] is 25 degrees of the Twins.

"And these are the learned men of Ishmael who observed the point of apogee of the sun, not all of whom lived in the same generation:

"The first was the Arabian scholar[2] . . . and Jahjâ ibn Abî Manṣûr[3] and el Merwadi[4] and Ibn al Mokaffa-a,[5] el Kufi,[6] Jacob al Kindi,[7] Thabit ibn Qora,[8] al Hakemi[9] the Hindu, Theon of Alexandria,[10] Ibrahim ez-Zarkali[11] the Spaniard, el-Batani,[12] Ibn Alostay,[13] and Ibn el-Alam.[14]

"And now I shall begin to translate the book of the Ishmael scholar.[15]

"Here beginneth the book of Muhammed ibn al Matani ibn Abdul Karasi[16] ibn Ali Ishmael explaining[17] the tables of al Khowarizmi. You remember what you saw in [the] tables of the planets, [namely,] mistakes and disagreements and [evidences] that their authors did not give any proof for what they told us to do, but they left it[18] to us, and presented them[19] as a matter of tradition without [any] discussion. In the case of a book of this kind the reader may attribute to its

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[1] The date of the work, seven years before Ibn Ezra's death.

[2] The name is omitted in the Hebrew text.

[3] He lived in the time of the caliph el-Mansûr. See the *Zeitschrift für Mathematik*, XII, 31, seq.; Suter's list, page 8.

[4] Steinschneider identifies him as possibly Merwesi or el Merwadsi, the Habas el Hâsib el-Merwazî of Suter's list. He lived at Bagdad in the time of el-Mâmûn.

[5] Not in Suter's list.

[6] Probably an error for el-Sûfî, that is, 'Abderrahmân ibn 'Omar, Abû'l-Hosein, el-Sûfî, who died in 986. His work on the fixed stars was translated into French in 1874. Possibly, however, Ibn Ezra means Muhammed ibn Zijâd ibn el-A'râbî of Kûfa, who wrote on astronomy as well as language, and who died about 846.

[7] For the translation of his works into Hebrew, see *Zeitschrift der Morgenl. Gesellsch.*, XVIII, 131, 181. For his work on Hindu arithmetic, see Woepcke, *Mém. sur la propagation*, 159.

[8] Thâbit ibn Qorra ibn Merwân, Abû'l-Hasan, el-Harrânî (826–901), one of the foremost Arab astronomers.

[9] Queried by Steinschneider. There were several Arab scholars of this name, but doubtless the caliph Hakem (996–1021) is meant, after whom the Hakem Tables composed by Ibn Junis were named. The words "the Hindu" are manifestly incorrect.

[10] Of course Theon was not one of the "learned men of Ishmael," so that Ibn Ezra uses the term rather loosely.

[11] Ibrâhîm el Zarquâla, or Zarqâlî, a famous Spanish instrument maker of the eleventh century. See *Zeitschrift für Math.*, XII, 34, 36; Steinschneider, *Etudes sur Zarkali*, Rome, 1884.

[12] Muhammed ibn Gâbir ibn Sinân, Abû Abdallâh, el-Battânî, a famous astronomer, known in Europe under the name Albatagnius. He died in 929.

[13] The name is doubtful.

[14] Probably the astronomer and astrologer 'Alî ibn el-Hosein, Abû'l Qâsim el-'Alawî, known under the name Ibn el-A'lam who died in 985.

[15] Beginning at this point, Steinschneider published only the Hebrew text of this fragment from Bodleian MS., Michael 400, without translation or comment. The rest of the text is still unpublished. See also the *Zeitung d. Morgenl. Gesellsch.*, XXV, p. 421.

[16] The transliteration is doubtful, Steinschneider gives Al Karuz. On el-Bîrûnî see the note on page 100.

The Codex Mich. 835 begins: "This book was written by Ahmed . . . elkerim for his brother Muhammed ben Ali ben Ishmael" etc.

[17] Literally, "about the reasons of."

[18] That is, the proving of the operations.

[19] That is, the rules.

author one of two things: either he does not himself understand the explanations, having merely learned the facts from someone, or he is jealous of his great wisdom and does not care to reveal it. We have already seen that scholars of undoubted wisdom did the same, as in the case of Al Khapash[1] in his work on grammar, known as Aloust, the result being that men learned in the science of grammar decided that his book was suited neither to a teacher nor to a student. You recall that you have found the same thing in al-Khowarizmi's tables and you attributed it to the condensed form[2] of the work or to the selfishness of the author.[3] You also remember that you found the work attributed to el-Ferġânî very far from perfect and even unsatisfactory for the uses to which you might wish to put it, and you saw that he explained things that are clear and are easy of comprehension while omitting all that was difficult and complicated, and you asked me for explanations so that nothing should be concealed from you. Therefore I explained all you asked for, and this will help your understanding and will satisfy any mathematician and man of science like yourself, and may God be my helper.

"Referring to what you have said about el-Ferġânî, [namely,] that you found his work far from perfect,—I have read it and have also found it so; but I have found in el-Ferġânî's book many things that showed him to be a wise man;[4] and the thought has occurred to me that el-Ferġânî had worked out in his mind the commentary on the book and the proofs [to be furnished], but death overtook him and he was unable to complete it. Now, however, someone has transcribed it and attributed to him [as complete] what still lacks the explanations. It is also possible that he completed his work before he died, but that part of it was lost, or that the book fell into the hands of some ignorant person who ruined its perfection, and so we cannot attribute these faults[5] to a lack of knowledge.[6] It is true that these things will not be understood, however, by a man who is ignorant of mathematics.[7] I have already written a book on selected topics of this work and have arranged it in gates[8] in the form of questions and answers, so that it explains more fully all that you may wish to know, and this is easier to understand and more convenient to study and remember, and I trust in the Creator that I may succeed in satisfying your wish. Question: Why did Muhammed al Khowarizmi say that to compute the Arabian month [it is necessary] to take the whole number of the Arabian year, write it in two different places, multiply one [number] by 354, retaining it; then multiplying the other by 13, dividing [the product] by 30, and then adding the quotient to the other result?"

This completes the translation of the Hebrew text as published by Stein-

[1] The name is doubtful. It is probably Akhfash, who died 830–835.
[2] Literally, shortness or brevity.
[3] That is, his jealousy lest others should share his wisdom.
[4] That is, that he was right.
[5] Literally, "things."
[6] That is, on the part of el-Ferġânî.
[7] That is, by such a man as evidently transcribed the work.
[8] *I. e.*, in chapters.

schneider from the Parma codex, and it covers Ibn Ezra's introduction. The text of el-Bîrûnî is in a manuscript in the Bodleian Library, and has never been published.

THE THEORY OF SIMILAR FIGURES.

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The object of this paper is to place in a new and perhaps a more satisfactory light the elements of the well-known theory of two and of three directly similar figures in a plane. The attempt has been made to keep in the foreground the idea of determining each of such a set of figures by a base-line, that is, by one of a set of homologous line-segments. This notion clarifies the conception, and appreciably simplifies the treatment; further, as shown below, it enables us to avoid certain false proofs which are to be found in the usual discussions of this subject. The use of directed angles[1] is again advantageous, but of course not essential.

The fundamental operations of elementary plane geometry are four in number:

(*a*) *Translation*, or motion of a figure such that every point is moved the same distance in the same direction,

(*b*) *Rotation* about a fixed point,

(*c*) *Reflection* with regard to a fixed line, which is the same in effect as turning the plane over on this line as axis,

(*d*) *Expansion* with regard to a fixed point, whereby the distance of each point of the figure from the fixed point is increased or decreased in a constant ratio (the same word *expansion* is used in all cases, whether the figure is actually enlarged or diminished).

If any figure is subjected to any succession of these operations, the resulting figure is similar to the original one; conversely it is easy to see that if two similar figures lie in a plane, one can be brought to coincide with the other by a combination, for instance, of a translation, a rotation, and an expansion. Our first object shall be to simplify this combination.

As a first lemma, we may note that if we operate on one of two directly similar figures in such a way that two of its points come to coincide with the corresponding points of the other, the figures coincide throughout.

Temporarily we shall use the word *homology* to designate the combination of a rotation and an expansion with regard to the same center.

THEOREM 1. *Given two line segments MN, $M'N'$, there exists a single operation, either a translation or a homology, that carries MN into $M'N'$.*

Case i. If MN and $M'N'$ are equal and parallel, and extend in the same direction, a translation that carries M into M' will carry N into N'.

Case ii. If $MNN'M'$ is a trapezoid, and MM' meets NN' at P, an expansion

[1] "Directed Angles in Elementary Geometry," R. A. Johnson, this MONTHLY, March, 1917, p. 101 ff.